ECE520 Course Notes

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1 Electromagnetics Fundamentals

1.1 Maxwell Equations

Integral Form	Differential Form
$\oint_C E \cdot dl = -\iint_s \frac{\partial B}{\partial t}$	$\nabla \times E = -\frac{\partial B}{\partial t}$
$\oint_C H \cdot dl = \iint_s \left(\frac{\partial D}{\partial t} + J\right) = I_{enc}$	$\nabla \times H = \frac{\partial D}{\partial t} + J$
$\oint_S D \cdot ds = \iiint \rho dV = Q$	$\nabla \cdot D = \rho$
$\oint_S B \cdot ds = 0$	$\nabla \cdot B = 0$

1.2 Lorentz Force

Charge q with velocity $\vec{v}$ through $\vec{B}$ field experiences force

$$F = q(\vec{v} \times \vec{B})$$

1.3 Force Charge Interactions

$$q\vec{v} = i\vec{l} \quad F = q(\vec{v} \times \vec{B}) \quad F = i(\vec{l} \times \vec{B})$$

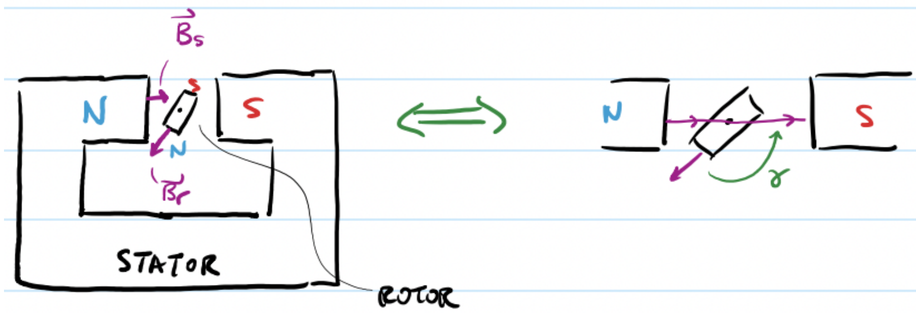
1.4 E Field

$$\vec{E} = \frac{\vec{F}}{q} \qquad \vec{E}_m = \vec{v} \times \vec{B}$$

1.5 Potential

$$e = -\int_a^b \vec{E} \cdot \vec{dl}$$

2 Generalized Machine Theory



$$T_q \propto |\vec{B}_r||\vec{B}_s| \sin \gamma$$

2.1 Flux

$$B_s = \frac{\phi_s}{A} \qquad A = \pi r l$$

2.2 Torque for Hypothetical Machine

$$T_q = \frac{2 i_r \phi_s l_r N}{\pi r l} = \frac{2N}{\pi} \phi_s i_r$$

2.3 Torque and Physical Parameters

$$T_q = \frac{2 N i_r}{r \pi} \cdot B_s \cdot \pi r^2 l$$

Where

- l is core length (axial length of rotor)
- r is rotor radius ($2r$ is core height)
- $\frac{2Ni_r}{r\pi}$ is linear current density on surface of rotor (limited by heat/cooling)
- B_s is stator B field (limited by magnetic saturation, magnetic properties)
- $\pi r^2 l$ is rotor volume

2.4 Torque Volume Relationship

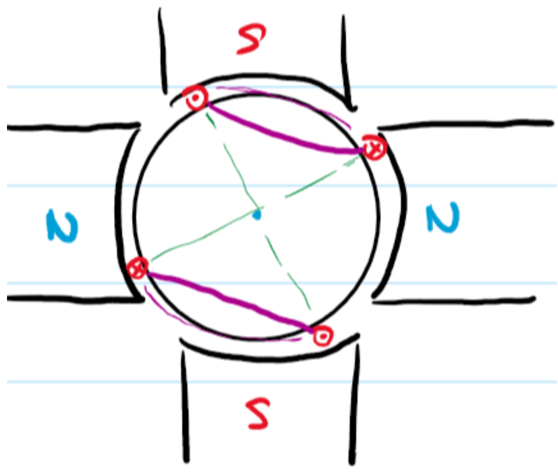
$$T_q \propto \textbf{Volume}$$

2.5 Power for Machines

$$P = T_q \omega_m$$

Valid for all DC/AC machines

2.6 Poles



Each pole produces flux:

$$T_q = \frac{pN}{\pi} \phi_s i_r$$

where p is the number of poles

2.7 Machine Constant

Define machine constant k:

$$k = \frac{pN}{\pi}$$

which yields

$$T_q = k \phi_s i_r$$

2.8 Speed Torque Relationships

When torque changes (has a ripple, or set point), speed is not always a linear function of torque:

$$\omega_m(t) = \frac{1}{J} \int T_q(\tau) d\tau$$

3 DC Machines

$$T_q = k\phi_s i_r \qquad k = \frac{pN}{\pi}$$
$$e_a = k\phi\omega_m$$

3.1 Torque

$$T_q = k\phi_s i_r$$

3.2 Mathematical Model

Electrical:

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ V_f = R_f i_f + L_f \frac{di_f}{dt} \end{cases}$$

Mechanical:

$$J\frac{d\omega_m}{dt} = T_q - T_{load} - T_{loss}$$

Coupling:

$$e_a = k\phi\omega_m \qquad k\phi = k_f i_f$$

3.3 Steady State Assumptions

$$\frac{di_a}{dt} = 0 \qquad \frac{d\omega_m}{dt} = 0 \qquad \frac{di_f}{dt} = 0$$

$$V_a = R_a I_a + e_a \qquad V_f = R_f I_f$$
$$e_a = k\phi\omega_m \qquad T_q = k\phi I_a$$
$$k\phi = k_f i_f$$
$$T_q = T_{load} + T_{loss}$$

Looks linear, but isnt

3.4 Common Load Characteristics

Typically, load is very non-linear (one of the below)

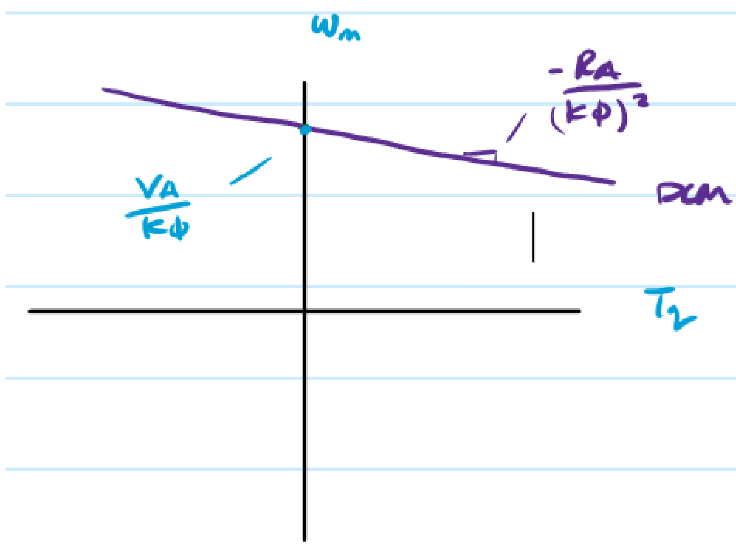
- Lifting: $T_{load} = c$
- Compressor: $T_{load} = c \cdot \omega_m$
- Fans/Pumps: $T_{load} = c \cdot \omega_m^2$
- Winders: $T_{load} = \frac{c}{\omega_m}$

3.5 Speed Torque Diagrams

$$V_a = R_a I_a + k\phi\omega_m$$

Set $I_a = \frac{T_q}{k\phi}$ and find:

$$\omega = \frac{V_a}{k\phi} - \frac{R_a}{(k\phi)^2} T_q$$



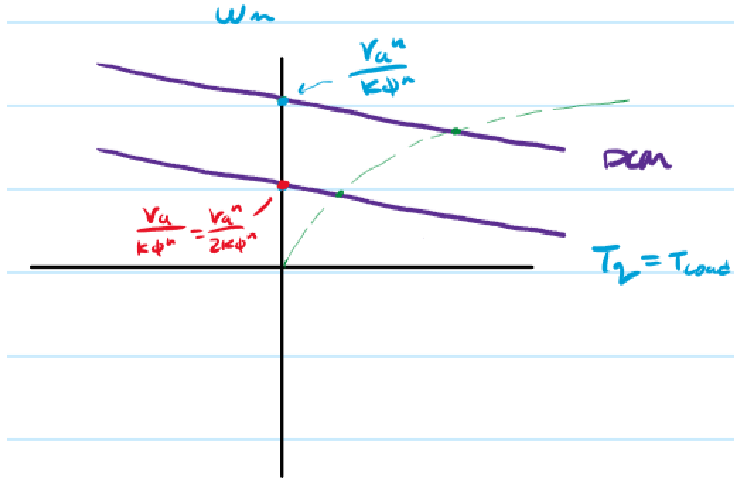
Usually, $-\frac{R_a}{(k\phi^2)}$ is shallow slope

3.6 Control Modes

3.6.1 V_a Control

Consider V_a^n and $k\phi^n$. Then change to $V_a = \frac{V_a}{m}$ to set V_a. Can use power electronics (buck) to supply variable V_a.

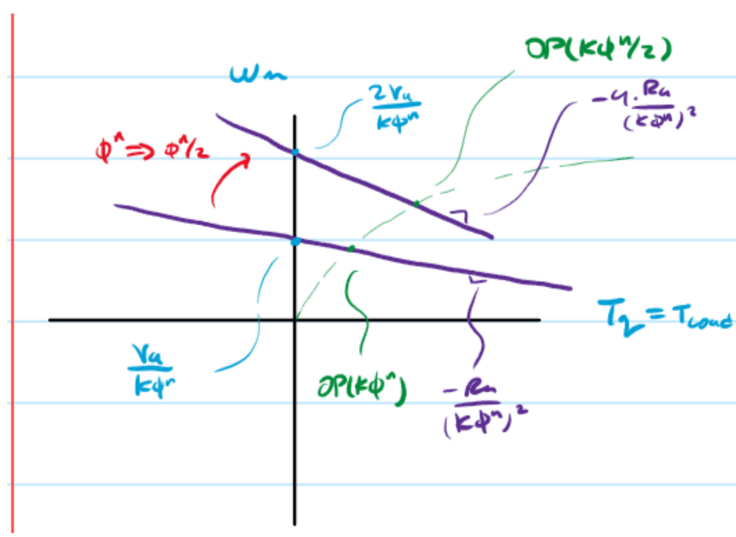
Can be **highly efficient** (DCM ¡1hp: $\eta > 90\%$, buck: $\eta > 95\%$



3.6.2 $k\phi$ Control

$$k\phi \propto i_f \qquad i_f = \frac{V_f}{R_f + R_{f,ex}}$$

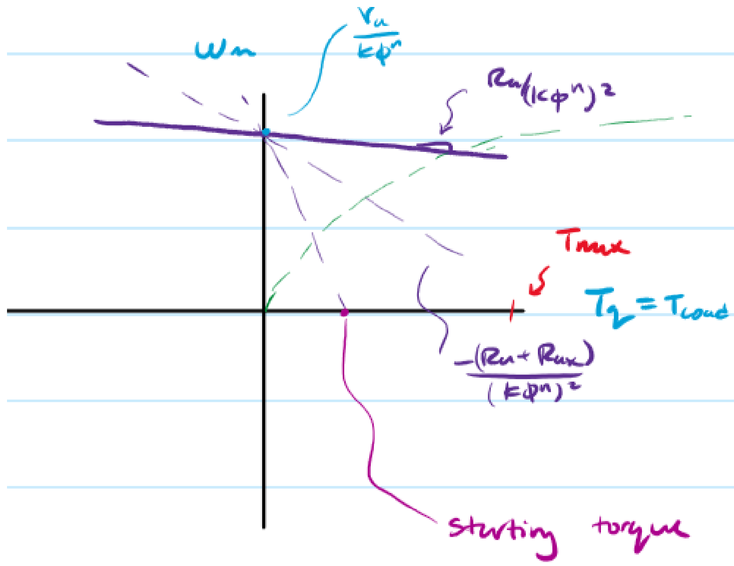
Add $R_{f,ex}$ in series with field winding to change $k\phi$



Caution: $T_q = k\phi i_a$. i_a can get huge

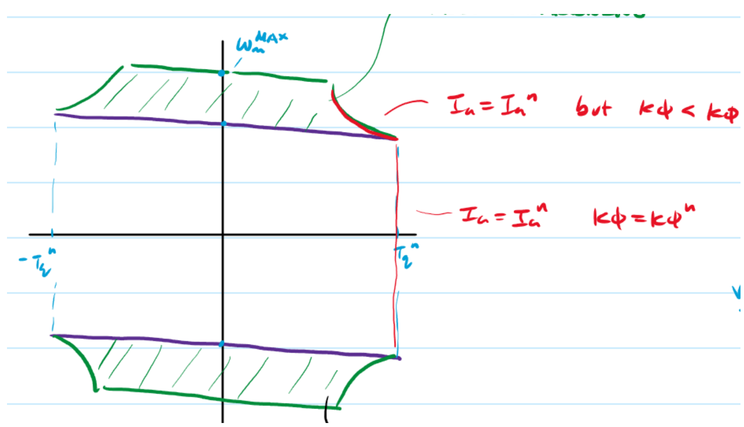
3.6.3 R_a Control

Add external resistor in series with R_a (induce large R_a)

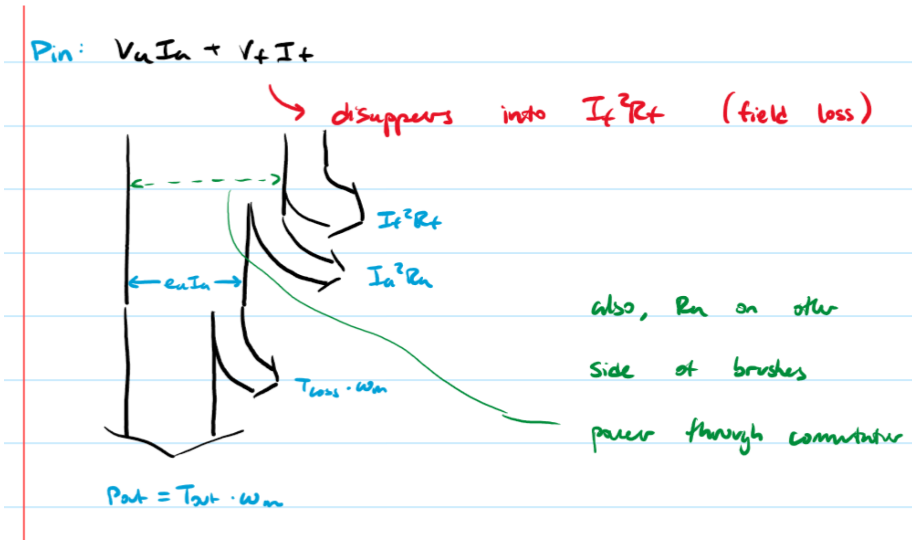


Don't want starting torque << T_{max} (torque at 0 speed should not be extremely large)

3.7 Operating Region

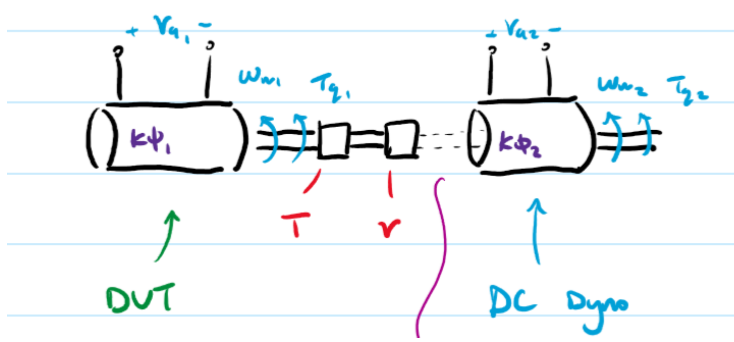


3.8 DCM Losses



4 Machine Testing

To test a motor, use a **dynamometer**.



Since shafts connected:

$$\omega_{m1} = \omega_{m2} = \omega_m$$

Need $J_{total}\frac{d\omega_m}{dt} = 0$ (neglecting loss):

$$T_{q_1} = T_{q_2} = 0 \qquad T_{q_1} = T_{q_2}$$

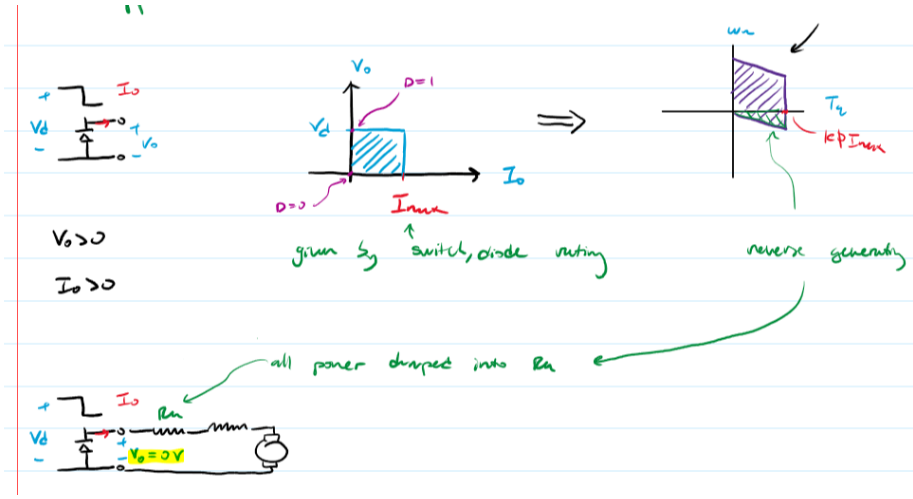
4.1 Servo Motor as Dyno

Can use speed control or torque control motor as Dyno (horizontal or vertical characteristic).

5 Power Electronics

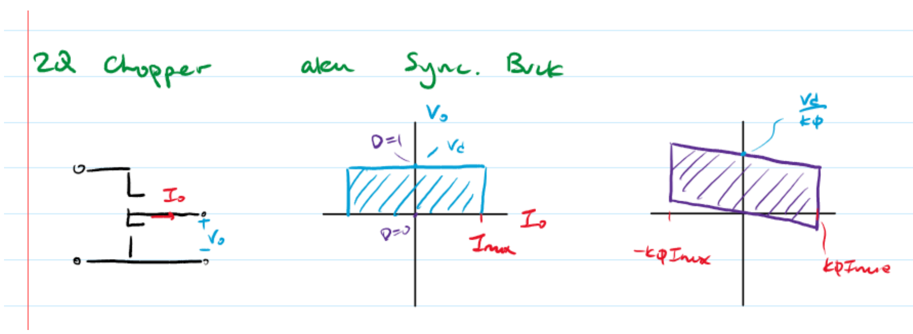
Machine Limits: $\omega_m - T_q$ plane Chopper limits: $V - I$ plane

5.1 1Q Chopper

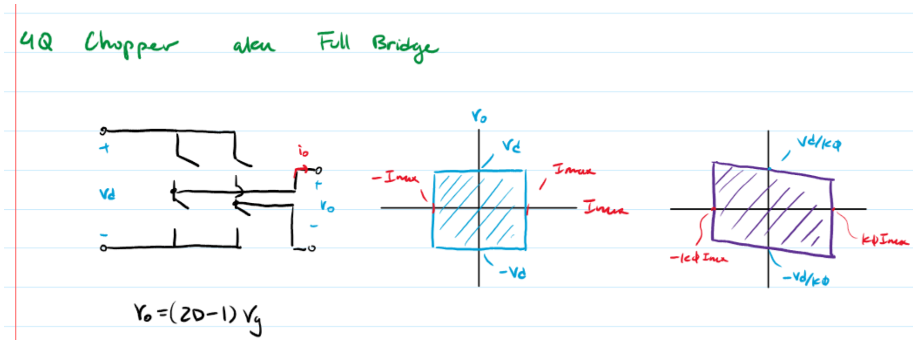


No regeneration!

5.2 2Q Chopper



5.3 4Q Chopper



Motor + Regen in both direction

- not frequently used
- only necessary when instantaneous forward/backwards transitions are required
- could also go backwards by inverting field (H-bridge)

6 DCM High Level Control

6.1 i_a Control

Set i_a, regulate torque (sometimes called torque control instead of current control). Create a vertical characteristic.

Observation: $i_a \to i_a^*$ changes very quickly, but ω_m changes slowly

6.1.1 1Q or 2Q Chopper

$$V_a = DV_d$$

Caution: very easy to go over-speed, need to ensure

$$k\phi = k\phi_{rated}$$

6.2 Control Block Diagrams

Convert DE into transfer functions. See 1.

6.3 Current Control Design

1. Track step changes in i_a^*
2. Reject e_a (assumed as DC)

3. Zero e_{ss} tracking error
4. Fast Response

Use a PI controller

$$C_i(s) = \frac{k_i(s + a_1)}{s}$$

6.4 Speed Control Design

1. Track step changes in ω^*
2. Reject constant $(T_{load} + T_{loss})$
3. Reasonably fast dynamics BUT must be slower than inner control loop

PI could work

$$C_\omega(s) = \frac{k_2(s + a_2)}{s}$$

Keep in mind, plant contains $\frac{1}{s}$.

6.5 Position Control

See 2.

Care: no one watching speed. Set i_a^* based on θ_m^* regulator

7 Space Vectors

$\vec{B_s}$ is a vector in 2D, used to express cylindrical coords in complex coordinates

$$\vec{B_s} = B_{sx}\hat{a_x} + B_{sy}\hat{a_y}$$

in **Space vector** form:

$$\vec{B_s} = |B_s|e^{j\angle B_s} \qquad |\vec{B_s}| - \sqrt{B_{sx}^2 + B_{sy}^2}$$

8 AC Synchronous Machines

Sizes from 1 mW to 1000 MW. Benefits compared to DCM:

- does not require commutator
- cheap
- able to run at $\approx 2 \cdot T_{max}$

8.1 Torque

$$T_q = 2N_s l \cdot r I_s |B_r| \sin\gamma$$

where

- l is core length (axial length of rotor)
- r is rotor radius

8.1.1 Torque Characteristics

For ripple free torque, require

1. const amplitude $|B_r|$
2. const amp $|B_s|$ in winding
3. smooth rotation of fictitious winding to create rotating South on stator for rotor North to follow

8.1.2 Synchronous Torque

$$T_q \propto \hat{I}_s |\hat{B}_r| \sin((\omega_s \zeta_s) - (\omega_{me} \zeta_{me}))$$

The sin component has an average value of zero UNLESS $\omega_s = \omega_{me}$ (synchronous speed equals mechanical speed).

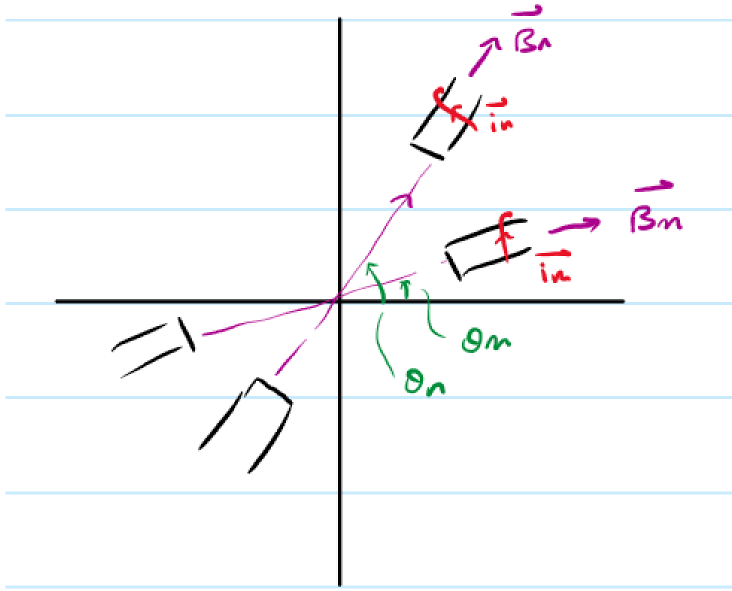
8.2 Orientation Matrix

$$\begin{bmatrix} B_{sx} \\ B_{sy} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} k \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

8.3 SM Space Vectors

$$\vec{B_s} = \begin{bmatrix} e^{j0} & e^{j\frac{\pi}{2}} \end{bmatrix} \begin{bmatrix} B_{s\alpha} \\ B_{s\beta} \end{bmatrix}$$

Where α is aligned with the x-axis, and β is aligned with the y-axis. In general,



$$\vec{B_s} = \begin{bmatrix} e^{j\theta_m} & e^{j\theta_n} \end{bmatrix} \begin{bmatrix} B_{sm} \\ B_{sn} \end{bmatrix}$$

8.3.1 Current Space Vector

Since we can't set B_s (only current controllable), defined **current space vector**

$$\vec{I_s} \equiv \frac{\vec{B_s}}{k}$$

and

$$B_{s\alpha} = kI_\alpha \qquad B_{s\beta} = kI_\beta$$

8.3.2 Voltage Space Vector

$$\vec{v} = \begin{bmatrix} e^{j0} & e^{j\frac{\pi}{2}} \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

Note:

- $\Re\{\vec{v}\}$ drives x-axis current, flux
- $\Im\{\vec{v}\}$ drives y-axis current, flux

8.4 2D Space Vector Modelling

$$R_\alpha = R_\beta = R \qquad L_\alpha = L_\beta = L$$

$$\vec{v} = R\vec{i} + L\frac{d\vec{i}}{dt}$$

where $\vec{v}, \vec{i}$ are the space vectors

8.4.1 Balanced Sinusoidal Steady State

For a 2ϕ machine, let

$$i_\alpha(t) = \hat{I}_s \cos(\omega_s t + \zeta_s)$$
$$i_\beta(t) = \hat{I}_s \sin(\omega_s t + \zeta_s)$$

Then

$$I_s(t) = \begin{bmatrix} 1 & j \end{bmatrix} \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \hat{I}_s e^{j(\omega_s t + \zeta_s)}$$

8.5 3D Space Vector Modelling

$$\vec{B_s} = \begin{bmatrix} e^{j0} & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \end{bmatrix} \begin{bmatrix} B_{sa} \\ B_{sb} \\ B_{sc} \end{bmatrix}$$

$$\begin{bmatrix} B_{sx} \\ B_{sy} \end{bmatrix} = \begin{bmatrix} 1 & \cos(\frac{2\pi}{3}) & \cos(\frac{4\pi}{3}) \\ 0 & \sin(\frac{2\pi}{3}) & \sin(\frac{4\pi}{3}) \end{bmatrix} \cdot \begin{bmatrix} B_{sa} \\ B_{sb} \\ B_{sc} \end{bmatrix}$$

8.5.1 Balanced Sinusoidal Steady State

For a 3ϕ machine, let

$$i_a(t) = \hat{I}_s \cos(\omega_s t + \zeta_A)$$
$$i_b(t) = \hat{I}_s \sin(\omega_s t + \zeta_B - \frac{2\pi}{3})$$
$$i_c(t) = \hat{I}_s \sin(\omega_s t + \zeta_s - \frac{4\pi}{3})$$

8.6 Clarke Transform

Orientation matrix not invertible (can't convert $\vec{B}$ to B_{sa}, B_{sb}, B_{sc}. Hence, use the **Clarke Transform**:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

where i_a, i_b, i_c are the 3ϕ current components. i_0 is the zero sequence current component (i_0 connected to neutral wire).

$\frac{2}{3}$ component for convenience s.t. $|\vec{I_s}| = \hat{I}_s$ in steady state (balanced current, $i_{a,b,c}$ has peak amplitude $\hat{I}_s$).

8.7 SM Modelling

$$V_\alpha = L_m \frac{di_\alpha}{dt} \qquad V_\beta = L_m \frac{di_\beta}{dt}$$

$$\vec{e_m} = L_m \frac{di_m}{dt}$$

$$\vec{i_m} = \vec{i_s} + \vec{i_f} \implies \vec{e_m} = L_m \frac{d\vec{i_s}}{dt} + L_m \frac{d\vec{i_f}}{dt}$$

If speed known (i.e. $\theta_{me} = \omega_{me} \cdot t + \rho_{me}$, then

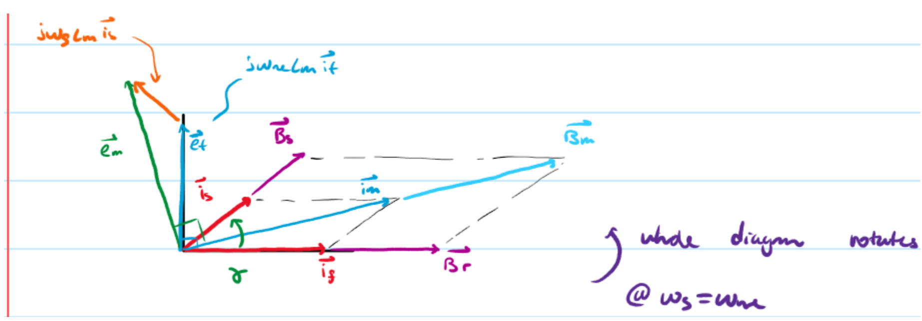
$$\vec{i_f} = k_f i_r e^{j(\omega_{me} \cdot t + \rho_{me})}$$

$$\frac{d\vec{i_f}}{dt} = j\omega_{me} \cdot k_f i_r e^{j(\omega_{me} \cdot t + \rho_{me})} = j\omega_{me} i_f$$

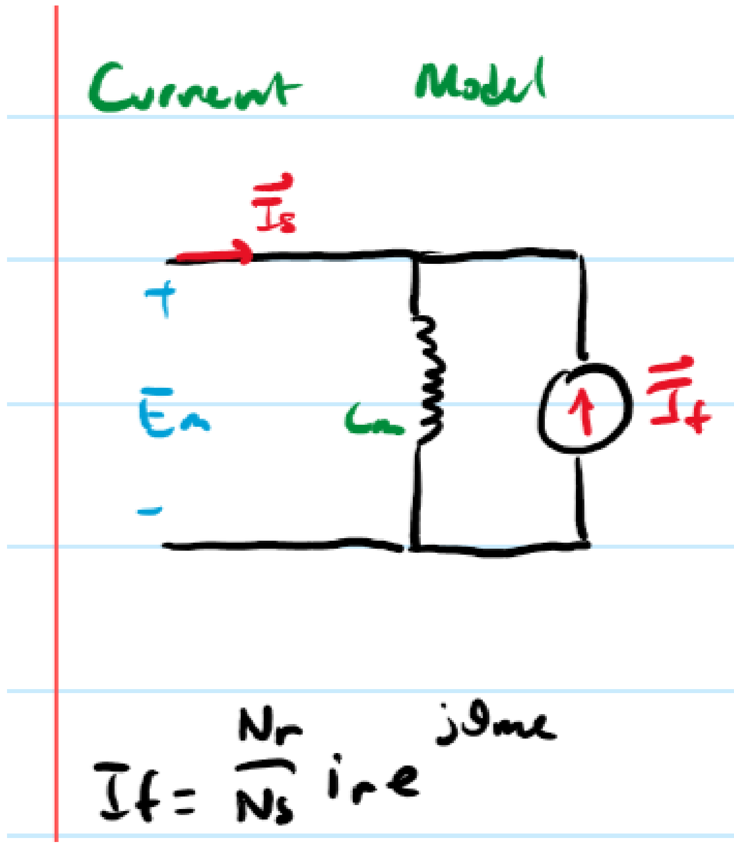
In summary,

$$\frac{d\vec{i_f}}{dt} = j\omega_{me} i_f \qquad \vec{e_f} = j\omega_{me} L_m \vec{i_f}$$

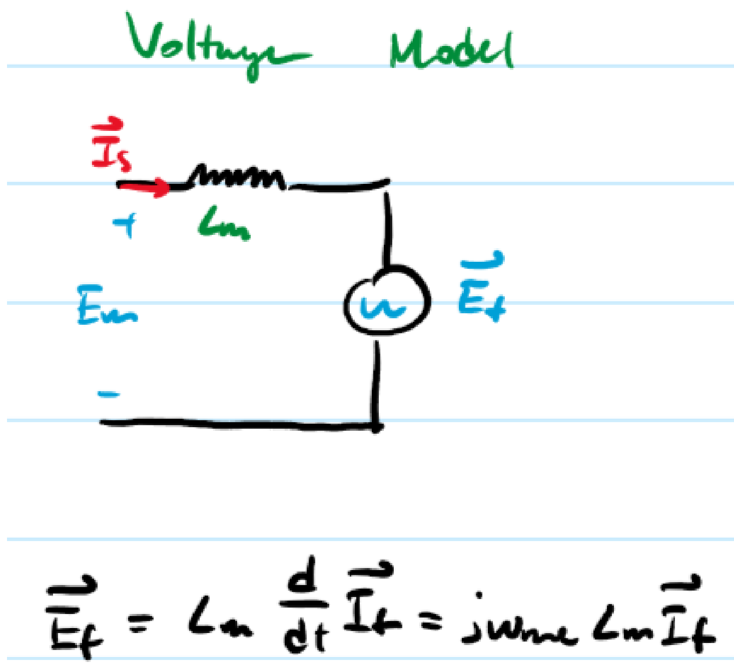
8.7.1 Vector Diagram



8.7.2 Current Model



8.7.3 Voltage Model



8.8 Multi Pole SM

Define:

- ω_s: $\vec{B}_s$ speed
- ω_{me}: $\vec{B}_r$ speed
- ω_m: physical shaft speed

$$\omega_{me} = \frac{p}{2}\omega_m \qquad \theta_{me} = \frac{p}{2}\theta_m$$
$$\omega_s = \omega_{me}$$

where p is the number of poles

8.9 Power

The power per phase is:

$$P_{perphase} = |\vec{E}_f||I_s|$$

Total power is

$$P = n \cdot \frac{1}{2}|\vec{E_f}||I_s| \sin\gamma$$

Apply $\vec{E_f} = j\omega_{me}L_m \cdot \vec{I_f}$

$$P = \frac{n}{2} j\omega_{me}L_m \cdot |I_f||I_s| \sin\gamma$$

where $\frac{1}{2}$ since per phase power is peak, not RMS. $\sin\gamma$ is the angle between $\vec{I_s}$ and $\vec{I_f}$.

Thus, we can write torque as

$$T_q = \frac{P}{\omega_m} = \frac{n}{2}\frac{p}{2} \cdot L_m|I_f||I_s| \sin\gamma$$

Define $k\phi = \frac{p}{2}L_m|I_f|$:

$$T_q = \frac{n}{2}k\phi|I_s| \sin\gamma$$

If $\gamma = 90$ ($\sin\gamma = 1$), then

$$T_q = T_{qmax} = \frac{n}{2}k\phi|I_s|$$

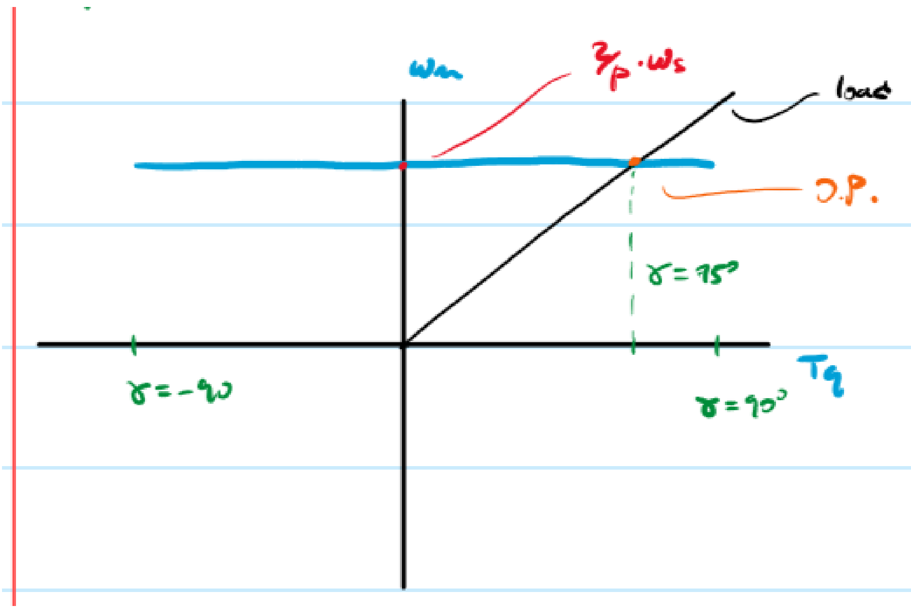
8.10 Brushless DCM

$$|\vec{E_f}| = \omega_{me} \cdot \frac{p}{2}L_m|I_f| = \omega_m k\phi$$

This resembles DC motor (but is AC synchronous), so sometimes called **Brushless DCM**.

8.11 Speed Torque Diagrams

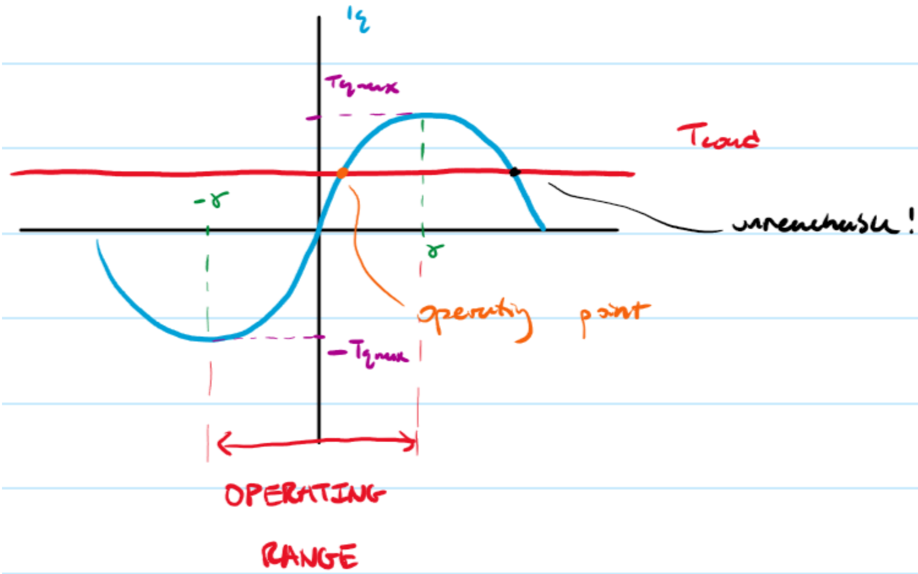
Constant Frequncy operation: Connect to fixed frequency ω_s voltage/current source



If operating point requires $\gamma > 90^0$, machine cannot reach (loss of synchronization).

8.12 γ-Torque Plot

Show $T_q \propto \gamma$



8.13 Voltage Source Operation

Assume

- Synchronous operation
- $\omega_{me} = \omega_s$
- Grid voltage known
- $\vec{V_g} = \vec{E_m}$

$$\vec{I_f} = k_f i_r e^{j\omega_{me}\cdot t}$$
$$\vec{E_f} = |E_f|e^{j\omega_s\cdot t+\frac{\pi}{2}} = j\omega_{me}L_m|\vec{I_f}|e^{j\omega_s\cdot t+\frac{\pi}{2}}$$

$$\vec{V_g} = \vec{E_m} = |\vec{E_m}|e^{j\omega_s\cdot t+\frac{\pi}{2}+\delta}$$

8.13.1 Power

$$P = \frac{n}{2}\cdot\Re\left\{\vec{E_f}\cdot\vec{I_s}^*\right\} = \frac{n}{2}\frac{\hat{E_f}\hat{E_m}}{\omega_s L_m}\sin\delta$$

8.13.2 Torque

$$T_q = \frac{n}{2}\frac{p}{2}\frac{1}{\omega_{me}}\frac{\hat{E_f}\hat{E_m}}{\omega_{me}L_m}\sin\delta$$
$$T_q = \frac{n}{2}\frac{p}{2}\frac{1}{\omega_{me}}|I_f|\frac{\hat{E_m}}{\omega_{me}L_m}\sin\delta$$

8.14 SM Parasitics

1. Stator Leakage Inductance L_s
2. Stator Resistance R_s
3. Rotational Losses
4. Mechanical Inertia

9 SM High Level Control

10 Induction Machines

No magnets on rotor or stator. Induce voltage in rotor from stator (induction). Requires **AC** voltage on stator.

Advantages:

- Low cost (no rare earth magnets required)
- No brushes (commutation)
- No DC field
- Low maintenance, robust
- High torque (can exceed T_{qmax})

Disadvantages:

- Heavier/bulkier than comparable PMSM
- High startup torque

10.1 IM Fields

$$\omega_s = \omega_m + \omega_r$$

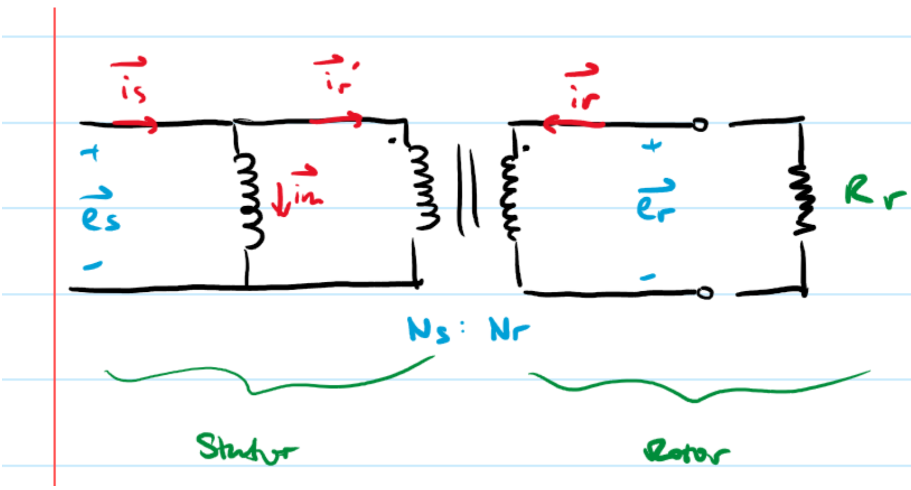
$$\omega_{me} = \frac{p}{2}\omega_m \qquad \omega_m = \frac{2}{p}\omega_{me}$$

10.2 IM Modelling

Assume $R_m = 0$. Define

$$N_1 i_m = \phi_m R_m \implies \phi_m = \frac{N_1 i_m}{R_m}$$

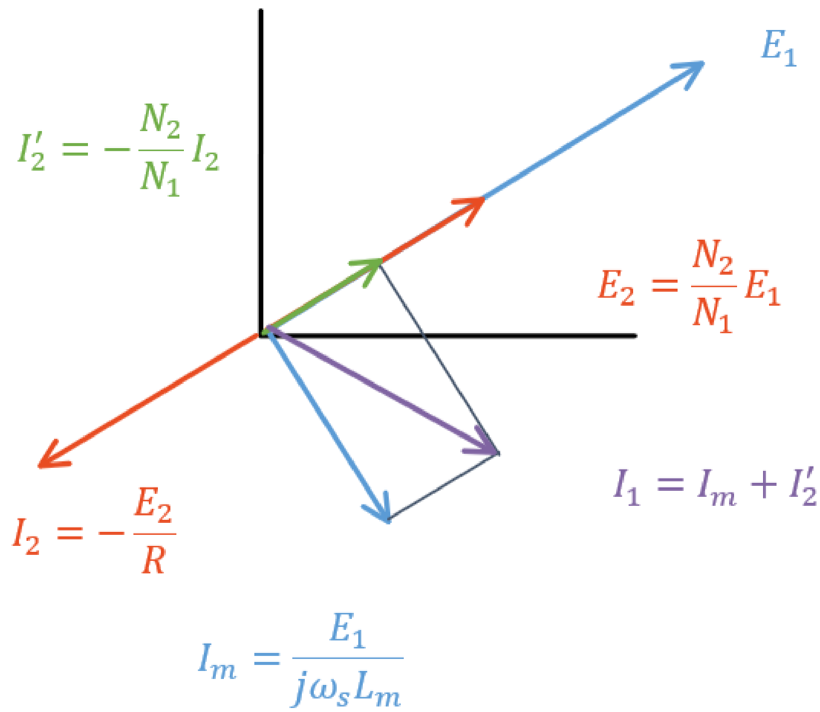
And set $\vec{e_s} = Ve^{j\omega_s t}$



Define **Magnetizing Inductance** L_m:

$$L_m = \frac{N_1^2}{R_m}$$

10.3 Phasor Diagram



10.4 Torque

$$T_q \propto |\vec{B_r}||\vec{B_s}| \sin\gamma \propto |\vec{I_r}||\vec{I_s}| \sin\gamma$$

If rotor load is resistive:

$$\vec{i_r} \perp \vec{i_m} \implies |\vec{i_s} \sin\gamma = |\vec{i_m}|$$
$$T_q \propto |\vec{i_r}||\vec{i_m}|$$

10.5 Rotating IM

Stator doesn't move, rotor rotates at ω_m

$$\vec{e_s} \vec{N_s} \cdot \frac{d\vec{\phi_m}}{dt} = L_m \cdot \frac{d\vec{i_m}}{dt}$$

voltage changes when rotor moves. For example, let $\vec{e_s} = \hat{E}_s e^{j\omega_s t}$. Then

$$\vec{\phi_m} = \frac{1}{N_s} \int \vec{e_s} dt = \vec{\phi_m}^{(s)}$$

$$\vec{e_r} = N_r \frac{d\phi_m}{dt}$$

10.5.1 IM Coordinate Systems

Define:

- $\vec{\phi_m}^{(r)}$: flux seen by rotor
- $\vec{\phi_m}^{(s)}$: flux seen by stator

$$\vec{\phi_m}^{(r)} = \vec{\phi_m}^{(s)} e^{-j\omega_m t}$$

10.6 Rotor Voltage, Flux

$$\vec{e_r}^{(r)} = N_r \frac{d\phi_m^{(r)}}{dt} = \frac{N_r}{N_s} \hat{E}_s \frac{\omega_s - \omega_m}{\omega_s} e^{j(\omega_s - \omega_m)t}$$

$$\vec{\phi_m}^{(r)} = \vec{\phi_m}^{(s)} e^{-j\omega_m t} = \frac{1}{N_s} \frac{\hat{E}_s}{j\omega_s} e^{j(\omega_s - \omega_m)t}$$

10.7 Slip

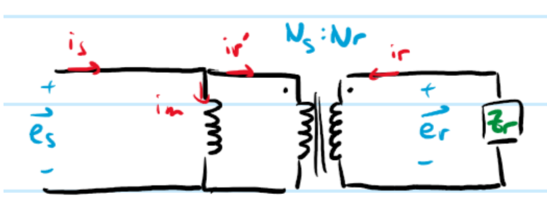
First, define ω_r to be the frequency of rotor voltage

$$\omega_r \omega_s - \omega_m$$

And **slip** to be:

$$S = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\omega_r}{\omega_s}$$

10.8 VI Relationships in an IM



$$\vec{e_r} = \frac{N_r}{N_s} \frac{\omega_r}{\omega_s} e^{-j\omega_m t} \vec{e_s}$$

$$\vec{i_r}' = -\frac{N_r}{N_s} e^{-j\omega_m t} \vec{i_r}$$

10.9 Power

$$P_{mech} = \frac{3}{2}\Re\left\{\vec{e_s}(\vec{i_r}')^*\right\} - \frac{3}{2}\Re\left\{\vec{e_r}(-\vec{i}_r)^*\right\}$$

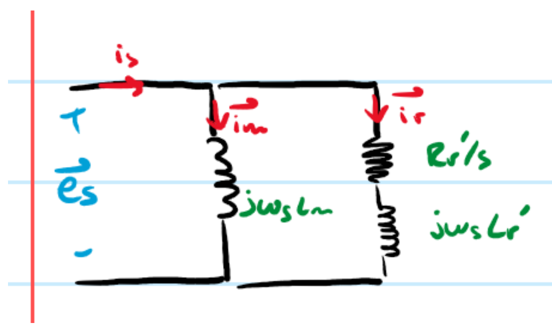
Where

- $\frac{3}{2}\Re\left\{\vec{e_s}(\vec{i_r}')^*\right\}$ is the power leaving stator
- $\frac{3}{2}\Re\left\{\vec{e_r}(-\vec{i}_r)^*\right\}$ is the power into Z_r

$$P_{mech} = \frac{3}{2}\left\{|\vec{i_r}'|^2\frac{R_r'}{S}\right\} - \frac{3}{2}\left\{|\vec{i_r'}|^2 R_r'\right\}$$

10.10 Reflecting Rotor Impedance

Reflecting both rotor impedance and resistance yields a new IM model:



$$\frac{R_r'}{S} = R_s + \left(\frac{1}{S} - 1\right)R_r'$$

Where

- $\frac{R_r'}{S}$ is the total effective resistance
- R_r is the real resistance (accounting for turns ratio)
- $\left(\frac{1}{S} - 1\right)R_r'$ is a representation of energy exchange (can be -ve or +ve) as a R value

10.11 Torque

$$T_q = \frac{P_{mech}}{\omega_m} = \frac{3}{2}|\vec{i_r}'|R_r'\frac{\frac{1}{S} - 1}{\omega_m}$$

The 3 represents 3ϕ (3 phase) IM.

$$T_q = \frac{3}{2}|\vec{i_r}'|\frac{R_r'}{\omega_r}$$

10.12 Speed Torque Diagram

$$T_q = \frac{3}{2}\frac{|\vec{e_s}|^2}{\omega_s^2}\frac{\omega_r}{R'_r}$$

Solving for ω_r

$$\omega_m = \omega_s - \frac{R'_r}{\left(\frac{\sqrt{3}}{\sqrt{2}}\frac{|\vec{e_s}|}{\omega_s}\right)^2} T_q$$

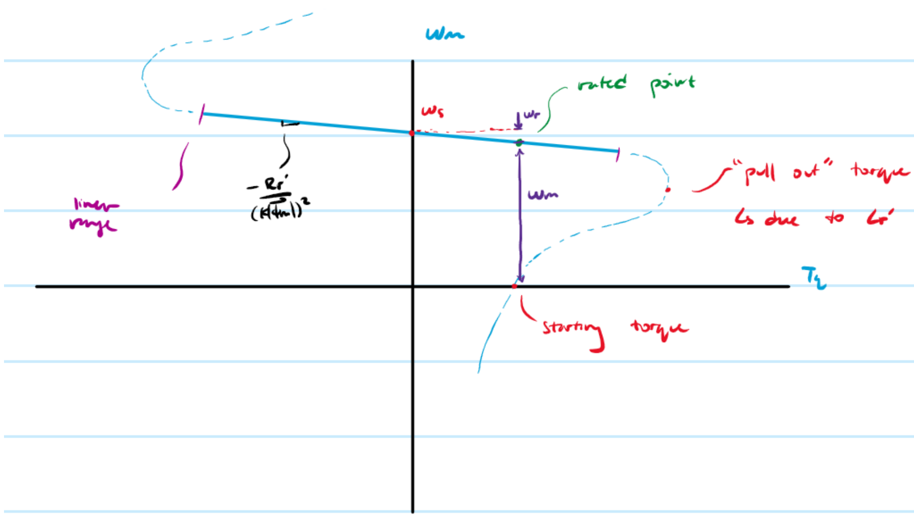
Where

$$k|vec\phi_m| = \frac{\sqrt{3}}{\sqrt{2}}\frac{|\vec{e_s}|}{\omega_s}$$

Rewriting:

$$\omega_m = \omega_s - \frac{R'_r}{(k|\vec{\phi_m}|^2}$$

which resembles the DCM speed torque relationship (w/ different intercept).



10.12.1 Linear Range

In the IM linear speed torque range, R'_r/s dominates $\vec{i_m} \perp \vec{i_r}$

$$T_q \propto |\vec{i_m}||\vec{i_r}|$$

10.12.2 Rated Flux

Rated flux occurs if

$$\frac{|\vec{e_s}|}{\omega_s} = \frac{|\vec{e_s}|_{rated}}{\omega_{s,rated}}$$

10.13 IM Nameplate

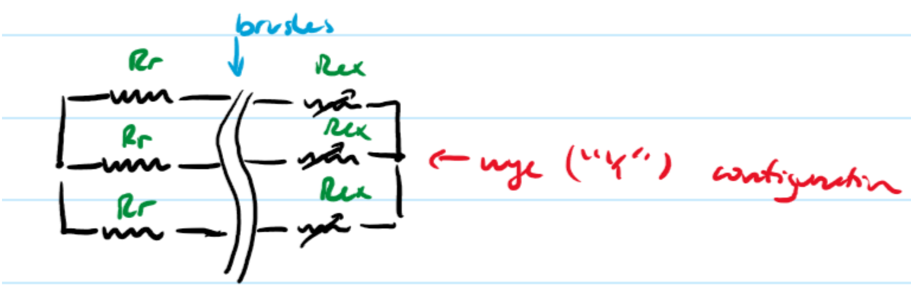
	3600	2 pole
N_m will be near	1800	4 pole
	1200	6 pole

where near means $\approx 95\%$

10.14 Control Modes

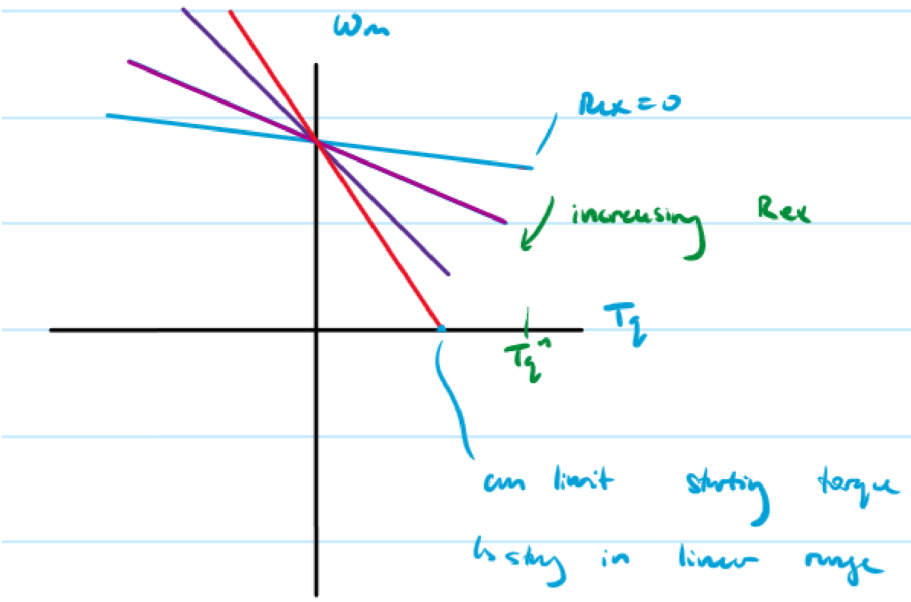
10.14.1 External Rotor Modes

requires doubly fed IM (rotor windings accessible)



R_{ex} should be balance in each phase

$$\omega_m = \omega_s - \frac{R'_r + R'_{ex}}{\left(\frac{\sqrt{3}}{\sqrt{2}} \frac{|\vec{e_s}|}{\omega_s}\right)^2} T_q$$



Pros:

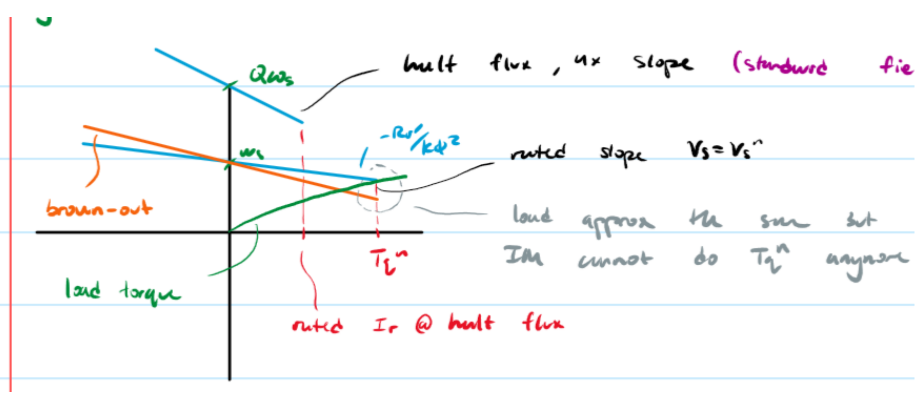
- can start high torque loads

Cons:

- R_{ex} losses
- Needs doubly fed IM (exposed rotor windings, brushes)

10.14.2 $k\phi$ Control

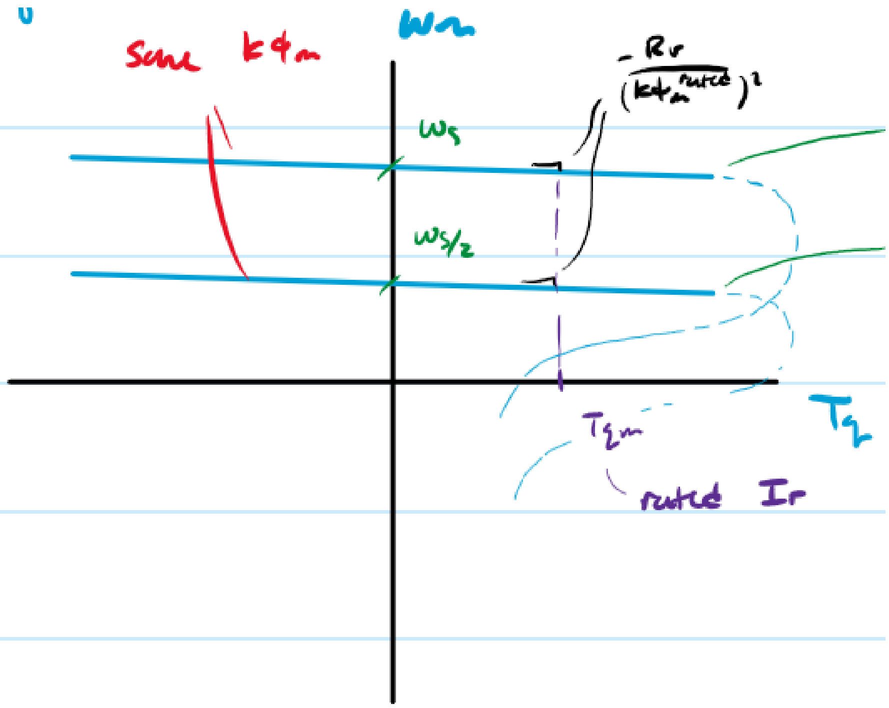
Standard field weakening (like in DCM)



10.14.3 V/f Control

Objectives;

- Constant flux (max allowable T_q)
- Varying intercept (ω_s)



Pros:

- Can achieve T_q^{rated} at any speed
- No R_{ex} losses
- use IM with internally shorted rotor
- no field current or PM's

Cons:

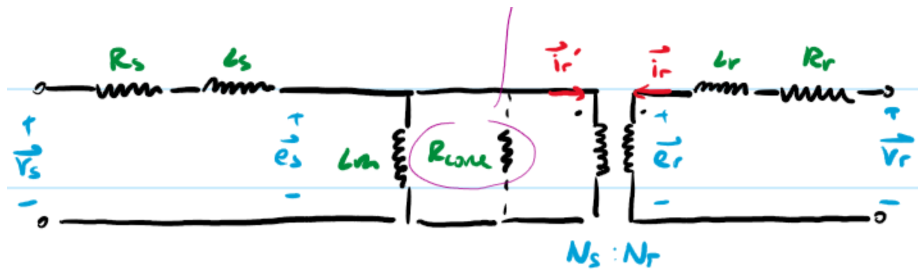
- Need inverter that can change both $|\vec{e_s}|$ and ω_s

V/f **control happens at** $\vec{e_s}$**, NOT** $\vec{v_s}$

10.15 IM Parasitics

1. Rotor Leakage Inductance L_r

2. Stator Resistance R_s
3. Stator Leakage Inductance L_s
4. Rotational Losses
5. Core Losses (Hysteresis, Eddy Currents)
6. Inertia



10.16 Pull-Out Torque

Absolute max T_q that machine can produce. Impacted predominantly by R_r, L_r

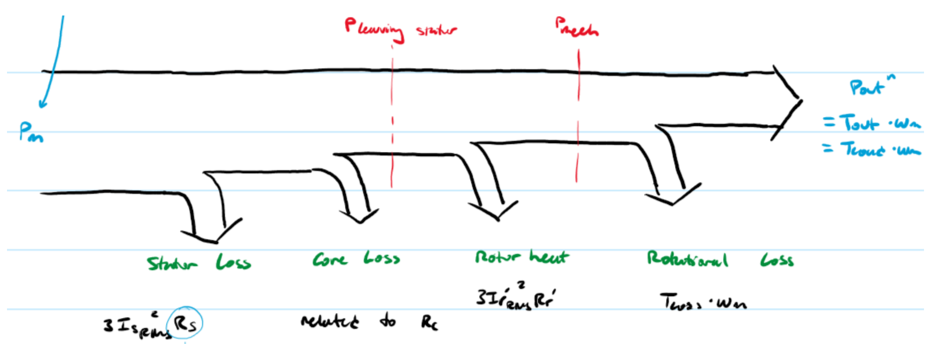
$$T_{q,po} = \frac{3}{2}\frac{|\vec{e_s}|^2}{\omega_s^2}\frac{1}{2L_r'}$$

Want a large linear range: want L_r small. But, this yields massive inrush currents.

10.17 Efficiency

$$P_m = \sqrt{3}|V_{s,llRMS}||I_{s,RMS}| \cdot PF$$

where PF is the power factor



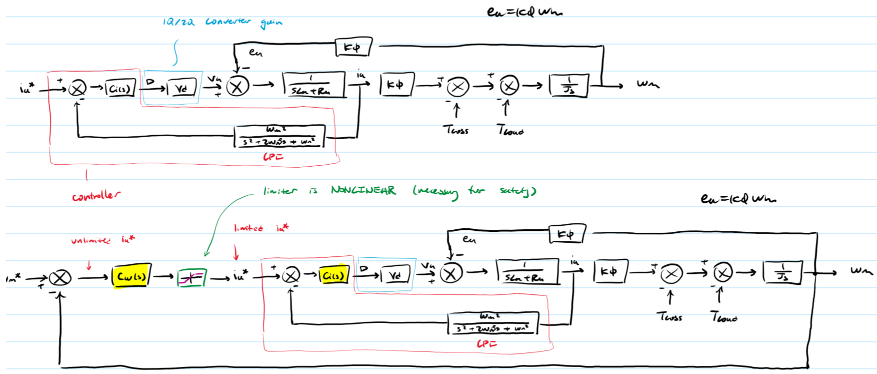


Figure 1: DC Current Control and Speed Control loops

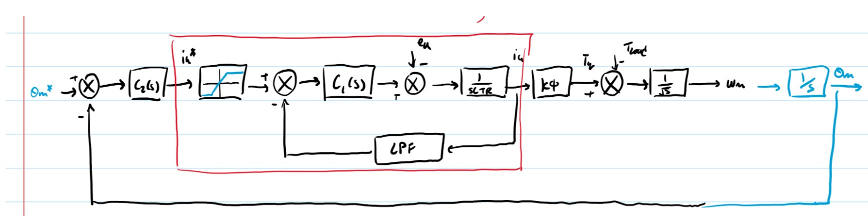


Figure 2: DC Position Control