

# 1 Electromagnetics Fundamentals

## 1.1 Maxwell Equations

Integral Form	Differential Form
$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\iint_S \frac{\partial B}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \left( \frac{\partial \mathbf{J}}{\partial t} + \mathbf{J} \right) = I_{enc}$	$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$
$\oint_S \mathbf{D} \cdot d\mathbf{s} = \iiint_V \rho dV = Q$	$\nabla \cdot \mathbf{D} = \rho$
$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	$\nabla \cdot \mathbf{B} = 0$

## 1.2 Lorentz Force

Charge  $q$  with velocity  $\vec{v}$  through  $\vec{B}$  field experiences force

$$\mathbf{F} = q(\vec{v} \times \vec{B})$$

## 1.3 Force Charge Interactions

$$q\vec{v} = i\vec{l} \quad \mathbf{F} = q(\vec{v} \times \vec{B}) \quad \mathbf{F} = i(\vec{l} \times \vec{B})$$

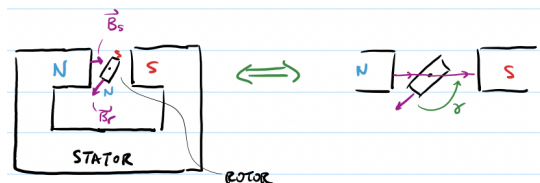
## 1.4 E Field

$$\vec{E} = \frac{\vec{F}}{q} \quad \vec{E}_m = \vec{v} \times \vec{B}$$

## 1.5 Potential

$$e = -\int_a^b \vec{E} \cdot d\vec{l}$$

## 2 Generalized Machine Theory



$$T_q \propto |\vec{B}_r| |\vec{B}_s| \sin \gamma$$

## 2.1 Flux

$$B_s = \frac{\phi_s}{A} \quad A = \pi r l$$

## 2.2 Torque for Hypothetical Machine

$$T_q = \frac{2i_r \phi_s l_r N}{\pi r l} = \frac{2N}{\pi} \phi_s i_r$$

## 2.3 Torque and Physical Parameters

$$T_q = \frac{2N i_r}{r \pi} \cdot B_s \cdot \pi r^2 l$$

Where

- $l$  is core length (axial length of rotor)
- $r$  is rotor radius ( $2r$  is core height)
- $\frac{2N i_r}{r \pi}$  is linear current density on surface of rotor (limited by heat/cooling)
- $B_s$  is stator  $B$  field (limited by magnetic saturation, magnetic properties)
- $\pi r^2 l$  is rotor volume

## 2.4 Torque Volume Relationship

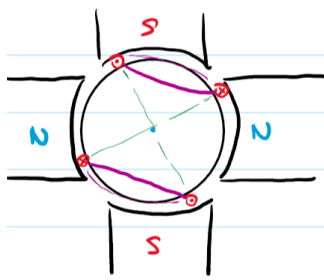
$$T_q \propto \text{Volume}$$

## 2.5 Power for Machines

$$P = T_q \omega_m$$

Valid for all DC/AC machines

## 2.6 Poles



Each pole produces flux:

$$T_q = \frac{pN}{\pi} \phi_s i_r$$

where  $p$  is the number of poles

## 2.7 Machine Constant

Define machine constant  $k$ :

$$k = \frac{pN}{\pi}$$

which yields

$$T_q = k \phi_s i_r$$

## 2.8 Speed Torque Relationships

When torque changes (has a ripple, or set point), speed is not always a linear function of torque:

$$\omega_m(t) = \frac{1}{J} \int T_q(\tau) d\tau$$

## 3 DC Machines

$$T_q = k \phi_s i_r \quad k = \frac{pN}{\pi}$$

$$e_a = k \phi \omega_m$$

## 3.1 Torque

$$T_q = k \phi_s i_r$$

## 3.2 Mathematical Model

Electrical:

$$\begin{cases} V_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \\ V_f = R_f i_f + L_f \frac{di_f}{dt} \end{cases}$$

Mechanical:

$$J \frac{d\omega_m}{dt} = T_q - T_{load} - T_{loss}$$

Coupling:

$$e_a = k \phi \omega_m \quad k \phi = k_f i_f$$

## 3.3 Steady State Assumptions

$$\frac{di_a}{dt} = 0 \quad \frac{d\omega_m}{dt} = 0 \quad \frac{di_f}{dt} = 0$$

$$V_a = R_a i_a + e_a \quad V_f = R_f i_f$$

$$e_a = k \phi \omega_m \quad T_q = k \phi I_a$$

$$k \phi = k_f i_f$$

$$T_q = T_{load} + T_{loss}$$

Looks linear, but isn't

## 3.4 Common Load Characteristics

Typically, load is very non-linear (one of the below)

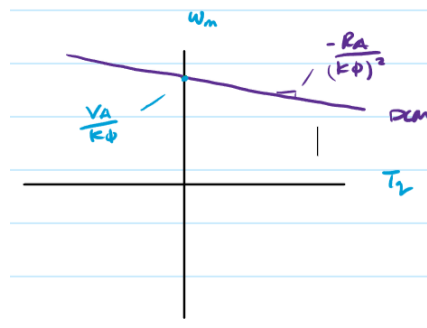
- Lifting:  $T_{load} = c$
- Compressor:  $T_{load} = c \cdot \omega_m$
- Fans/Pumps:  $T_{load} = c \cdot \omega_m^2$
- Winders:  $T_{load} = \frac{c}{\omega_m}$

## 3.5 Speed Torque Diagrams

$$V_a = R_a i_a + k \phi \omega_m$$

Set  $I_a = \frac{T_q}{k \phi}$  and find:

$$\omega = \frac{V_a}{k \phi} - \frac{R_a}{(k \phi)^2} T_q$$



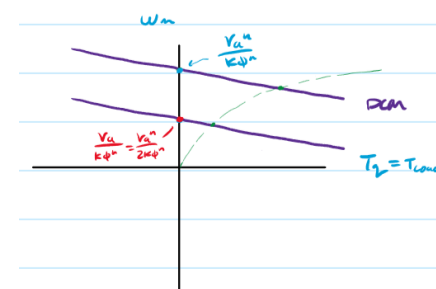
Usually,  $-\frac{R_a}{(k \phi)^2}$  is shallow slope

## 3.6 Control Modes

### 3.6.1 $V_a$ Control

Consider  $V_a^n$  and  $k \phi^n$ . Then change to  $V_a = \frac{V_a^n}{m}$  to set  $V_a$ . Can use power electronics (buck) to supply variable  $V_a$ .

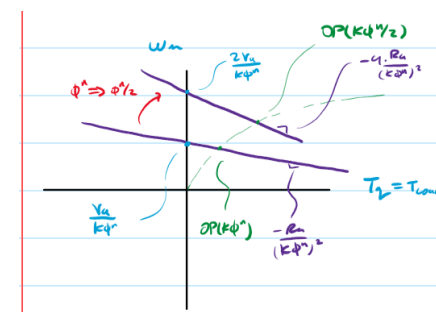
Can be **highly efficient** (DCM < 1hp:  $\eta > 90\%$ , buck:  $\eta > 95\%$ )



### 3.6.2 $k \phi$ Control

$$k \phi \propto i_f \quad i_f = \frac{V_f}{R_f + R_{f,ex}}$$

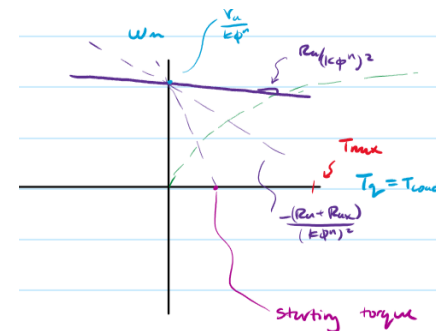
Add  $R_{f,ex}$  in series with field winding to change  $k \phi$



Caution:  $T_q = k \phi i_a$ .  $i_a$  can get huge

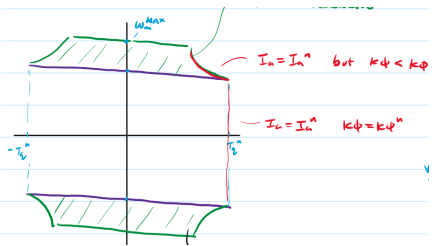
### 3.6.3 $R_a$ Control

Add external resistor in series with  $R_a$  (induce large  $R_a$ )

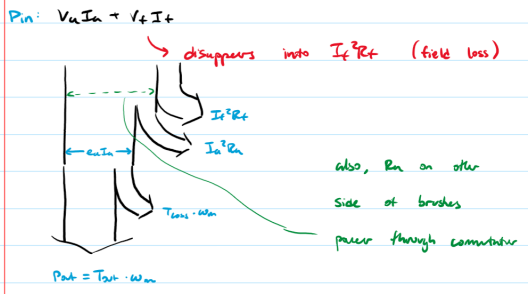


Don't want starting torque  $\ll T_{max}$  (torque at 0 speed should not be extremely large)

### 3.7 Operating Region

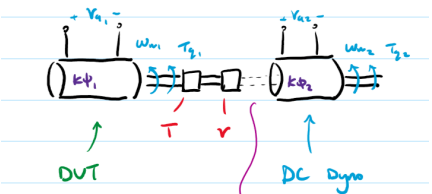


### 3.8 DCM Losses



### 4 Machine Testing

To test a motor, use a dynamometer.



Since shafts connected:

$$\omega_{m1} = \omega_{m2} = \omega_m$$

Need  $J_{total} \frac{d\omega_m}{dt} = 0$  (neglecting loss):

$$T_{q1} = T_{q2} = 0 \quad T_{q1} = T_{q2}$$

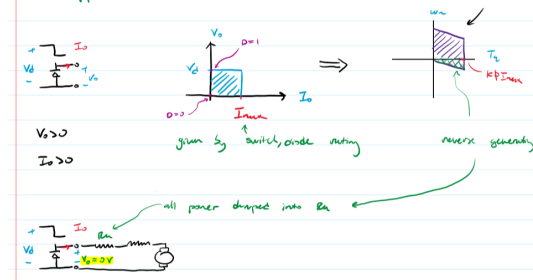
### 4.1 Servo Motor as Dyno

Can use speed control or torque control motor as Dyno (horizontal or vertical characteristic).

### 5 Power Electronics

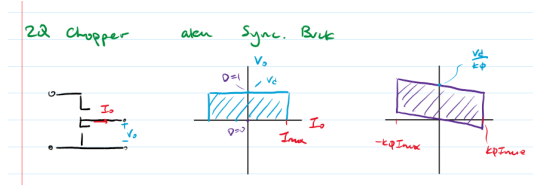
Machine Limits:  $\omega_m - T_q$  plane Chopper limits:  $V - I$  plane

### 5.1 1Q Chopper

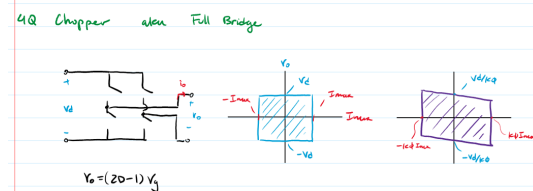


No regeneration!

### 5.2 2Q Chopper



### 5.3 4Q Chopper



Motor + Regen in both direction

- not frequently used
- only necessary when instantaneous forward/backwards transitions are required
- could also go backwards by inverting field (H-bridge)

### 6 DCM High Level Control

#### 6.1 $i_a$ Control

Set  $i_a^*$ , regulate torque (sometimes called torque control instead of current control). Create a vertical characteristic. Observation:  $i_a \rightarrow i_a^*$  changes very quickly, but  $\omega_m$  changes slowly

#### 6.1.1 1Q or 2Q Chopper

$$V_a = DV_d$$

Caution: very easy to go over-speed, need to ensure

$$k\phi = k\phi_{rated}$$

#### 6.2 Control Block Diagrams

Convert DE into transfer functions. See 1.

#### 6.3 Current Control Design

1. Track step changes in  $i_a^*$
2. Reject  $e_a$  (assumed as DC)
3. Zero  $e_{ss}$  tracking error

### 4. Fast Response

Use a PI controller

$$C_i(s) = \frac{k_i(s + a_1)}{s}$$

### 6.4 Speed Control Design

1. Track step changes in  $\omega^*$
2. Reject constant ( $T_{load} + T_{loss}$ )
3. Reasonably fast dynamics BUT must be slower than inner control loop

PI could work

$$C_\omega(s) = \frac{k_2(s + a_2)}{s}$$

Keep in mind, plant contains  $\frac{1}{s}$ .

### 6.5 Position Control

See 2.

Care: no one watching speed. Set  $i_a^*$  based on  $\theta_m^*$  regulator

### 7 Space Vectors

$\vec{B}_s$  is a vector in 2D, used to express cylindrical coords in complex coordinates

$$\vec{B}_s = B_{sx} \hat{a}_x + B_{sy} \hat{a}_y$$

in Space vector form:

$$\vec{B}_s = |B_s| e^{j\angle B_s} \quad |\vec{B}_s| = \sqrt{B_{sx}^2 + B_{sy}^2}$$

### 8 AC Synchronous Machines

Sizes from 1 mW to 1000 MW. Benefits compared to DCM:

- does not require commutator
- cheap
- able to run at  $\approx 2 \cdot T_{max}$

#### 8.1 Torque

$$T_q = 2N_s l \cdot r I_s |B_r| \sin \gamma$$

where

- $l$  is core length (axial length of rotor)
- $r$  is rotor radius

#### 8.1.1 Torque Characteristics

For ripple free torque, require

1. const amplitude  $|B_r|$
2. const amp  $|B_s|$  in winding
3. smooth rotation of fictitious winding to create rotating South on stator for rotor North to follow

#### 8.1.2 Synchronous Torque

$$T_q \propto \hat{I}_s |\hat{B}_r| \sin((\omega_s \zeta_s) - (\omega_{me} \zeta_{me}))$$

The sin component has an average value of zero UNLESS  $\omega_s = \omega_{me}$  (synchronous speed equals mechanical speed).

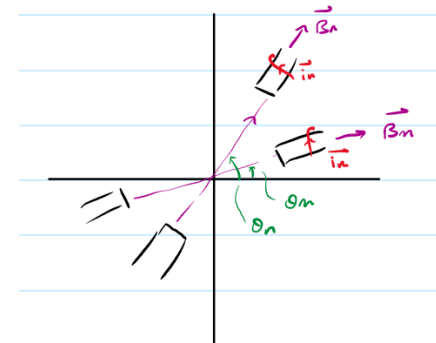
### 8.2 Orientation Matrix

$$\begin{bmatrix} B_{sx} \\ B_{sy} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} k \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

### 8.3 SM Space Vectors

$$\vec{B}_s = [e^{j0} \quad e^{j\frac{\pi}{2}}] \begin{bmatrix} B_{s\alpha} \\ B_{s\beta} \end{bmatrix}$$

Where  $\alpha$  is aligned with the x-axis, and  $\beta$  is aligned with the y-axis. In general,



$$\vec{B}_s = [e^{j\theta_m} \quad e^{j\theta_n}] \begin{bmatrix} B_{sm} \\ B_{sn} \end{bmatrix}$$

#### 8.3.1 Current Space Vector

Since we can't set  $B_s$  (only current controllable), defined current space vector

$$\vec{I}_s \equiv \frac{\vec{B}_s}{k}$$

and

$$B_{s\alpha} = k I_\alpha \quad B_{s\beta} = k I_\beta$$

#### 8.3.2 Voltage Space Vector

$$\vec{v} = [e^{j0} \quad e^{j\frac{\pi}{2}}] \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$

Note:

- $\Re\{\vec{v}\}$  drives x-axis current, flux
- $\Im\{\vec{v}\}$  drives y-axis current, flux

#### 8.4 2D Space Vector Modelling

$$R_\alpha = R_\beta = R \quad L_\alpha = L_\beta = L$$

$$\vec{v} = R \vec{i} + L \frac{d\vec{i}}{dt}$$

where  $\vec{v}, \vec{i}$  are the space vectors

#### 8.4.1 Balanced Sinusoidal Steady State

For a  $2\phi$  machine, let

$$i_\alpha(t) = \hat{I}_s \cos(\omega_s t + \zeta_s)$$

$$i_\beta(t) = \hat{I}_s \sin(\omega_s t + \zeta_s)$$

Then

$$I_s(t) = [1 \quad j] \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} = \hat{I}_s e^{j(\omega_s t + \zeta_s)}$$

### 8.5 3D Space Vector Modelling

$$\vec{B}_s = \begin{bmatrix} e^{j0} & e^{j\frac{2\pi}{3}} & e^{j\frac{4\pi}{3}} \end{bmatrix} \begin{bmatrix} B_{sa} \\ B_{sb} \\ B_{sc} \end{bmatrix}$$

$$\begin{bmatrix} B_{sx} \\ B_{sy} \end{bmatrix} = \begin{bmatrix} 1 & \cos(\frac{2\pi}{3}) & \cos(\frac{4\pi}{3}) \\ 0 & \sin(\frac{2\pi}{3}) & \sin(\frac{4\pi}{3}) \end{bmatrix} \cdot \begin{bmatrix} B_{sa} \\ B_{sb} \\ B_{sc} \end{bmatrix}$$

#### 8.5.1 Balanced Sinusoidal Steady State

For a 3φ machine, let

$$i_a(t) = \hat{I}_s \cos(\omega_s t + \zeta_A)$$

$$i_b(t) = \hat{I}_s \sin(\omega_s t + \zeta_B - \frac{2\pi}{3})$$

$$i_c(t) = \hat{I}_s \sin(\omega_s t + \zeta_C - \frac{4\pi}{3})$$

#### 8.6 Clarke Transform

Orientation matrix not invertible (can't convert  $\vec{B}$  to  $B_{sa}, B_{sb}, B_{sc}$ ). Hence, use the **Clarke Transform**:

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

where  $i_a, i_b, i_c$  are the 3φ current components.  $i_0$  is the zero sequence current component ( $i_0$  connected to neutral wire).

$\frac{2}{3}$  component for convenience s.t.  $|\vec{I}_s| = \hat{I}_s$  in steady state (balanced current,  $i_{a,b,c}$  has peak amplitude  $\hat{I}_s$ ).

#### 8.7 SM Modelling

$$V_\alpha = L_m \frac{di_\alpha}{dt} \quad V_\beta = L_m \frac{di_\beta}{dt}$$

$$\vec{e}_m = L_m \frac{di_m}{dt}$$

$$\vec{i}_m = \vec{i}_s + \vec{i}_f \implies \vec{e}_m = L_m \frac{d\vec{i}_s}{dt} + L_m \frac{d\vec{i}_f}{dt}$$

If speed known (i.e.  $\theta_{me} = \omega_{me} \cdot t + \rho_{me}$ ), then

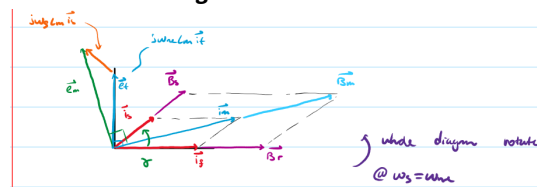
$$\vec{i}_f = k_f i_r e^{j(\omega_{me} t + \rho_{me})}$$

$$\frac{d\vec{i}_f}{dt} = j\omega_{me} \cdot k_f i_r e^{j(\omega_{me} t + \rho_{me})} = j\omega_{me} \vec{i}_f$$

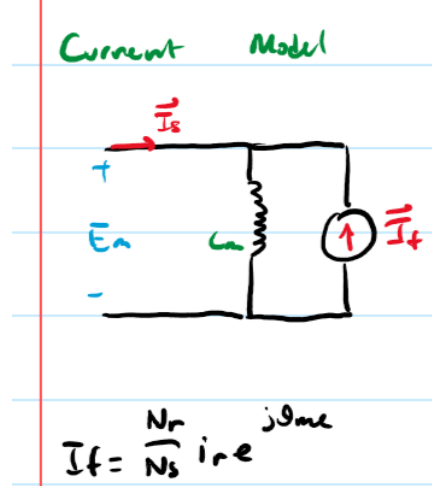
In summary,

$$\frac{d\vec{i}_f}{dt} = j\omega_{me} \vec{i}_f \quad \vec{e}_f = j\omega_{me} L_m \vec{i}_f$$

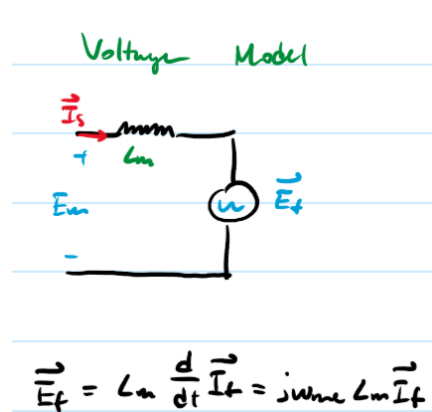
#### 8.7.1 Vector Diagram



#### 8.7.2 Current Model



#### 8.7.3 Voltage Model



#### 8.8 Multi Pole SM

Define:

- $\omega_s$ :  $\vec{B}_s$  speed
- $\omega_{me}$ :  $\vec{B}_r$  speed
- $\omega_m$ : physical shaft speed

$$\omega_{me} = \frac{p}{2} \omega_m \quad \theta_{me} = \frac{p}{2} \theta_m$$

$$\omega_s = \omega_{me}$$

where  $p$  is the number of poles

#### 8.9 Power

The power per phase is:

$$P_{per\ phase} = |\vec{E}_f| |I_s|$$

Total power is

$$P = n \cdot \frac{1}{2} |\vec{E}_f| |I_s| \sin \gamma$$

Apply  $\vec{E}_f = j\omega_{me} L_m \cdot \vec{I}_f$

$$P = \frac{n}{2} j\omega_{me} L_m \cdot |I_f| |I_s| \sin \gamma$$

where  $\frac{1}{2}$  since per phase power is peak, not RMS.

$\sin \gamma$  is the angle between  $\vec{I}_s$  and  $\vec{I}_f$ .

Thus, we can write torque as

$$T_q = \frac{P}{\omega_m} = \frac{n}{2} \cdot \frac{p}{2} \cdot L_m |I_f| |I_s| \sin \gamma$$

Define  $k\phi = \frac{p}{2} L_m |I_f|$ :

$$T_q = \frac{n}{2} k\phi |I_s| \sin \gamma$$

If  $\gamma = 90$  ( $\sin \gamma = 1$ ), then

$$T_q = T_{qmax} = \frac{n}{2} k\phi |I_s|$$

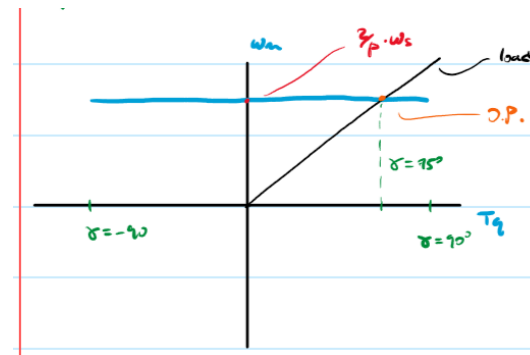
#### 8.10 Brushless DCM

$$|\vec{E}_f| = \omega_{me} \cdot \frac{p}{2} L_m |I_f| = \omega_m k\phi$$

This resembles DC motor (but is AC synchronous), so sometimes called **Brushless DCM**.

#### 8.11 Speed Torque Diagrams

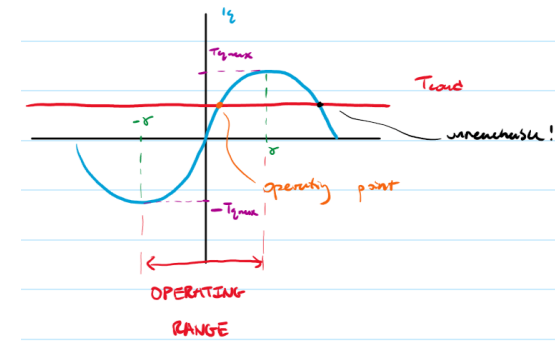
Constant Frequency operation: Connect to fixed frequency  $\omega_s$  voltage/current source



If operating point requires  $\gamma > 90^\circ$ , machine cannot reach (loss of synchronization).

#### 8.12 $\gamma$ -Torque Plot

Show  $T_q \propto \gamma$



#### 8.13 Voltage Source Operation

Assume

- Synchronous operation
- $\omega_{me} = \omega_s$
- Grid voltage known
- $\vec{V}_g = \vec{E}_m$

$$\vec{I}_f = k_f i_r e^{j\omega_{me} t}$$

$$\vec{E}_f = |E_f| e^{j\omega_s t + \frac{\pi}{2}} = j\omega_{me} L_m |\vec{I}_f| e^{j\omega_s t + \frac{\pi}{2}}$$

$$\vec{V}_g = \vec{E}_m = |\vec{E}_m| e^{j\omega_s t + \frac{\pi}{2} + \delta}$$

#### 8.13.1 Power

$$P = \frac{n}{2} \cdot \Re \{ \vec{E}_f \cdot \vec{I}_s^* \} = \frac{n}{2} \frac{\hat{E}_f \hat{E}_m}{\omega_s L_m} \sin \delta$$

#### 8.13.2 Torque

$$T_q = \frac{n}{2} \frac{p}{2} \frac{1}{\omega_{me}} \frac{\hat{E}_f \hat{E}_m}{\omega_{me} L_m} \sin \delta$$

$$T_q = \frac{n}{2} \frac{p}{2} \frac{1}{\omega_{me}} |I_f| \frac{\hat{E}_m}{\omega_{me} L_m} \sin \delta$$

#### 8.14 SM Parasitics

- Stator Leakage Inductance  $L_s$
- Stator Resistance  $R_s$
- Rotational Losses
- Mechanical Inertia

### 9 SM High Level Control

#### 10 Induction Machines

No magnets on rotor or stator. Induce voltage in rotor from stator (induction). Requires AC voltage on stator.

Advantages:

- Low cost (no rare earth magnets required)
- No brushes (commutation)
- No DC field
- Low maintenance, robust
- High torque (can exceed  $T_{qmax}$ )

Disadvantages:

- Heavier/bulkier than comparable PMSM
- High startup torque

### 10.1 IM Fields

$$\omega_s = \omega_m + \omega_r$$

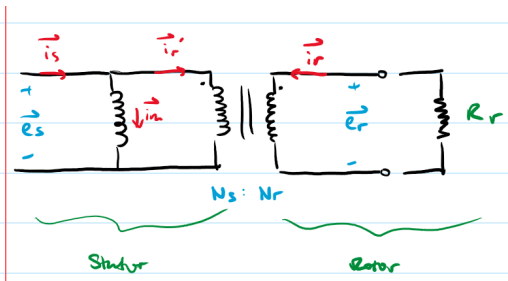
$$\omega_{me} = \frac{p}{2} \omega_m \quad \omega_m = \frac{2}{p} \omega_{me}$$

### 10.2 IM Modelling

Assume  $R_m = 0$ . Define

$$N_1 i_m = \phi_m R_m \implies \phi_m = \frac{N_1 i_m}{R_m}$$

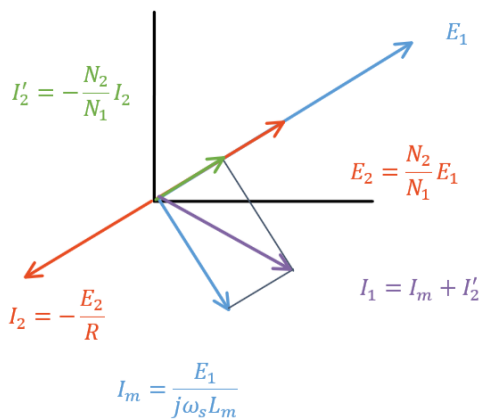
And set  $\vec{e}_s = V e^{j\omega_s t}$



Define Magnetizing Inductance  $L_m$ :

$$L_m = \frac{N_1^2}{R_m}$$

### 10.3 Phasor Diagram



### 10.4 Torque

$$T_q \propto |\vec{B}_r| |\vec{B}_s| \sin \gamma \propto |\vec{i}_r| |\vec{i}_s| \sin \gamma$$

If rotor load is resistive:

$$\vec{i}_r \perp \vec{i}_m \implies |\vec{i}_s \sin \gamma| = |\vec{i}_m|$$

$$T_q \propto |\vec{i}_r| |\vec{i}_m|$$

### 10.5 Rotating IM

Stator doesn't move, rotor rotates at  $\omega_m$

$$\vec{e}_s N_s \cdot \frac{d\vec{\phi}_m}{dt} = L_m \cdot \frac{d\vec{i}_m}{dt}$$

voltage changes when rotor moves. For example, let  $\vec{e}_s = \hat{E}_s e^{j\omega_s t}$ . Then

$$\vec{\phi}_m = \frac{1}{N_s} \int \vec{e}_s dt = \vec{\phi}_m^{(s)}$$

$$\vec{e}_r = N_r \frac{d\phi_m}{dt}$$

### 10.5.1 IM Coordinate Systems

Define:

- $\vec{\phi}_m^{(r)}$ : flux seen by rotor

- $\vec{\phi}_m^{(s)}$ : flux seen by stator

$$\vec{\phi}_m^{(r)} = \vec{\phi}_m^{(s)} e^{-j\omega_m t}$$

### 10.6 Rotor Voltage, Flux

$$\vec{e}_r^{(r)} = N_r \frac{d\phi_m^{(r)}}{dt} = \frac{N_r}{N_s} \hat{E}_s \frac{\omega_s - \omega_m}{\omega_s} e^{j(\omega_s - \omega_m)t}$$

$$\vec{\phi}_m^{(r)} = \vec{\phi}_m^{(s)} e^{-j\omega_m t} = \frac{1}{N_s} \frac{\hat{E}_s}{j\omega_s} e^{j(\omega_s - \omega_m)t}$$

### 10.7 Slip

First, define  $\omega_r$  to be the frequency of rotor voltage

$$\omega_r := \omega_s - \omega_m$$

And slip to be:

$$S = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\omega_r}{\omega_s}$$

### 10.8 VI Relationships in an IM



$$\vec{e}_r = \frac{N_r}{N_s} \frac{\omega_r}{\omega_s} e^{-j\omega_m t} \vec{e}_s$$

$$\vec{i}_r' = -\frac{N_r}{N_s} e^{-j\omega_m t} \vec{i}_r$$

### 10.9 Power

$$P_{mech} = \frac{3}{2} \Re \left\{ \vec{e}_s (\vec{i}_r')^* \right\} - \frac{3}{2} \Re \left\{ \vec{e}_r (-\vec{i}_r)^* \right\}$$

Where

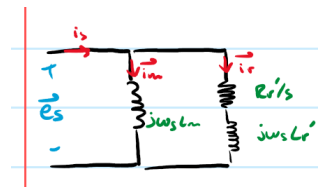
- $\frac{3}{2} \Re \left\{ \vec{e}_s (\vec{i}_r')^* \right\}$  is the power leaving stator

- $\frac{3}{2} \Re \left\{ \vec{e}_r (-\vec{i}_r)^* \right\}$  is the power into  $Z_r$

$$P_{mech} = \frac{3}{2} \left\{ |\vec{i}_r'|^2 \frac{R_r'}{S} \right\} - \frac{3}{2} \left\{ |\vec{i}_r|^2 R_r' \right\}$$

### 10.10 Reflecting Rotor Impedance

Reflecting both rotor impedance and resistance yields a new IM model:



$$\frac{R_r'}{S} = R_s + \left( \frac{1}{S} - 1 \right) R_r'$$

Where

- $\frac{R_r'}{S}$  is the total effective resistance

- $R_r$  is the real resistance (accounting for turns ratio)

- $\left( \frac{1}{S} - 1 \right) R_r'$  is a representation of energy exchange (can be -ve or +ve) as a  $R$  value

### 10.11 Torque

$$T_q = \frac{P_{mech}}{\omega_m} = \frac{3}{2} |\vec{i}_r'| |R_r' \frac{1}{S} - 1| \frac{1}{\omega_m}$$

The 3 represents 3 $\phi$  (3 phase) IM.

$$T_q = \frac{3}{2} |\vec{i}_r'| \frac{R_r'}{\omega_r}$$

### 10.12 Speed Torque Diagram

$$T_q = \frac{3}{2} \frac{|\vec{e}_s|^2}{\omega_s^2} \frac{\omega_r}{R_r'}$$

Solving for  $\omega_r$

$$\omega_m = \omega_s - \frac{R_r'}{\left( \frac{\sqrt{3}}{\sqrt{2}} \frac{|\vec{e}_s|}{\omega_s} \right)^2} T_q$$

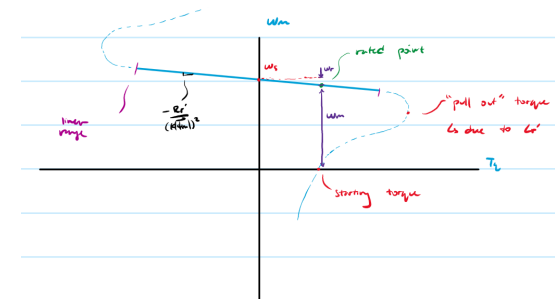
Where

$$k |\text{vec} \phi_m| = \frac{\sqrt{3}}{\sqrt{2}} \frac{|\vec{e}_s|}{\omega_s}$$

Rewriting:

$$\omega_m = \omega_s - \frac{R_r'}{(k|\vec{\phi}_m|)^2}$$

which resembles the DCM speed torque relationship (w/ different intercept).



### 10.12.1 Linear Range

In the IM linear speed torque range,  $R_r'/s$  dominates  $\vec{i}_m \perp \vec{i}_r$

$$T_q \propto |\vec{i}_m| |\vec{i}_r|$$

### 10.12.2 Rated Flux

Rated flux occurs if

$$\frac{|\vec{e}_s|}{\omega_s} = \frac{|\vec{e}_s|_{\text{rated}}}{\omega_{s,\text{rated}}}$$

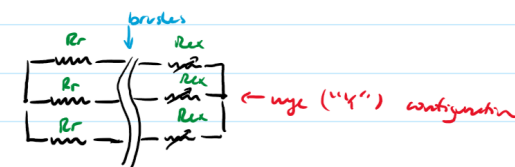
### 10.13 IM Nameplate

$N_m$ will be near	3600	2 pole
	1800	4 pole
	1200	6 pole

where near means  $\approx 95\%$

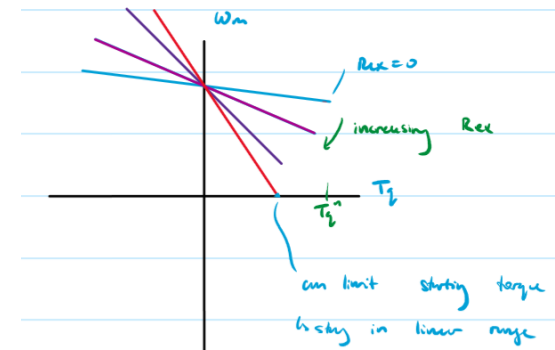
### 10.14 Control Modes

**10.14.1 External Rotor Modes** requires doubly fed IM (rotor windings accessible)



$R_{ex}$  should be balance in each phase

$$\omega_m = \omega_s - \frac{R_r' + R_{ex}'}{\left( \frac{\sqrt{3}}{\sqrt{2}} \frac{|\vec{e}_s|}{\omega_s} \right)^2} T_q$$



Pros:

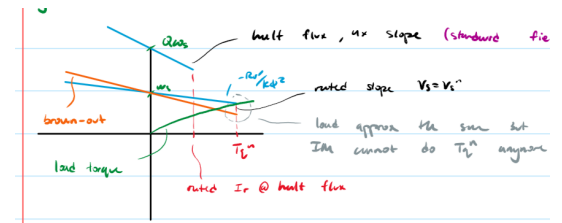
- can start high torque loads

Cons:

- $R_{ex}$  losses
- Needs doubly fed IM (exposed rotor windings, brushes)

**10.14.2  $k\phi$  Control**

Standard field weakening (like in DCM)

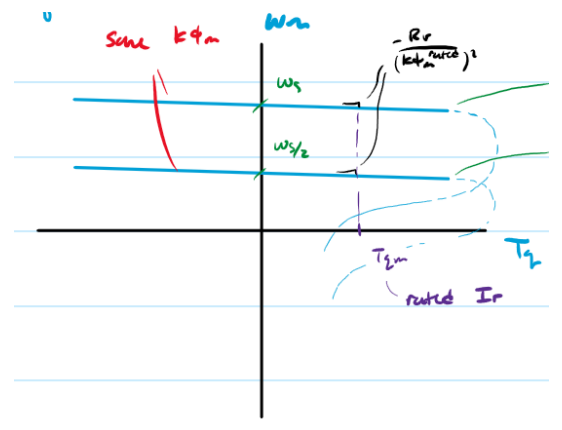


**10.14.3 V/f Control**

Objectives;

- Constant flux (max allowable  $T_q$ )

- Varying intercept ( $\omega_s$ )



Pros:

- Can achieve  $T_q^{rated}$  at any speed
- No  $R_{ex}$  losses

- use IM with internally shorted rotor
- no field current or PM's

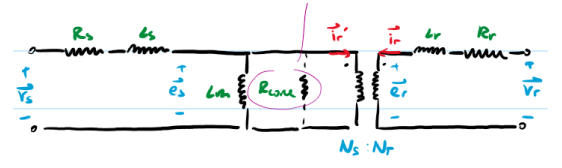
Cons:

- Need inverter that can change both  $|\vec{e}_s|$  and  $\omega_s$

V/f control happens at  $\vec{e}_s$ , NOT  $\vec{v}_s$

**10.15 IM Parasitics**

1. Rotor Leakage Inductance  $L_r$
2. Stator Resistance  $R_s$
3. Stator Leakage Inductance  $L_s$
4. Rotational Losses
5. Core Losses (Hysteresis, Eddy Currents)
6. Inertia



**10.16 Pull-Out Torque**

Absolute max  $T_q$  that machine can produce. Impacted predominantly by  $R_r, L_r$

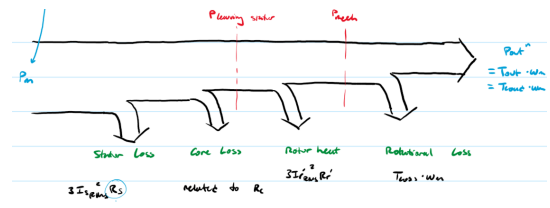
$$T_{q,po} = \frac{3}{2} \frac{|\vec{e}_s|^2}{\omega_s^2} \frac{1}{2L_r'}$$

Want a large linear range: want  $L_r$  small. But, this yields massive inrush currents.

**10.17 Efficiency**

$$P_m = \sqrt{3} |V_{s,RMS}| |I_{s,RMS}| \cdot PF$$

where PF is the power factor



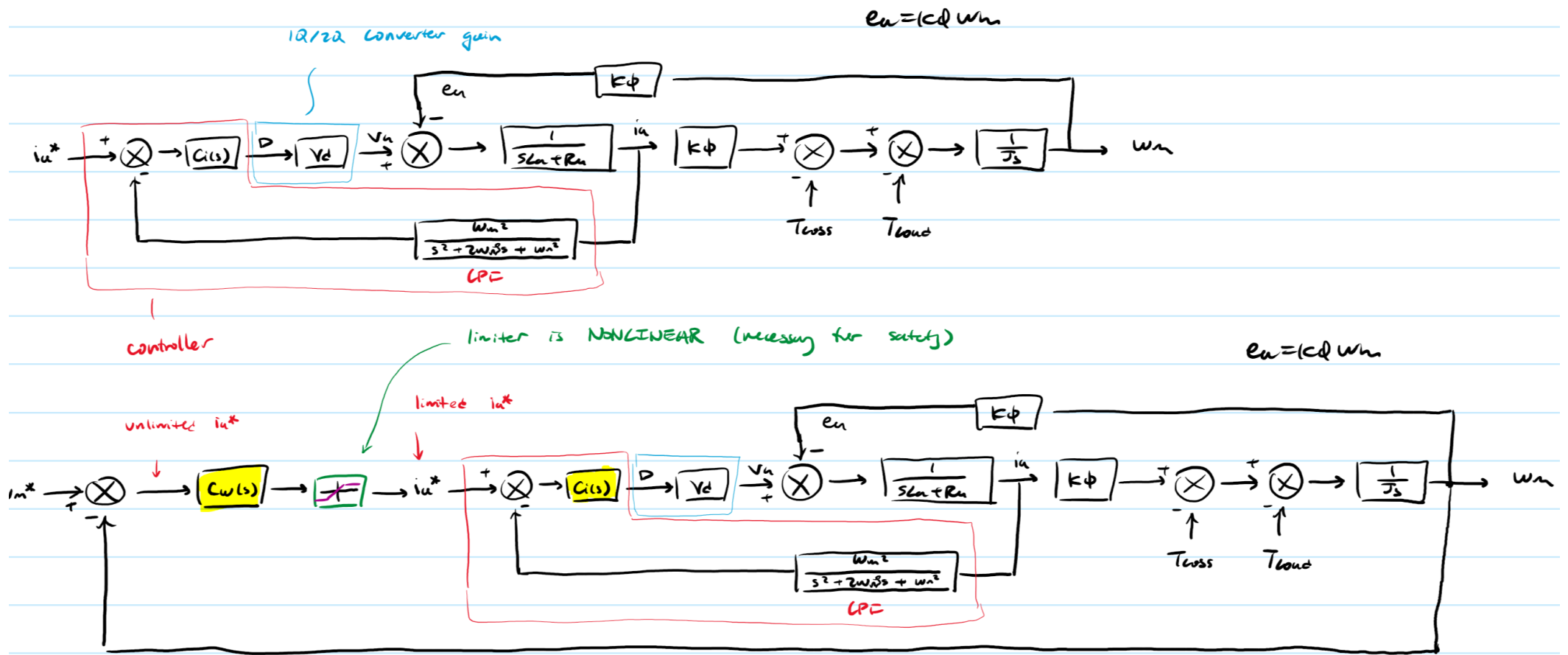


Figure 1: DC Current Control and Speed Control loops

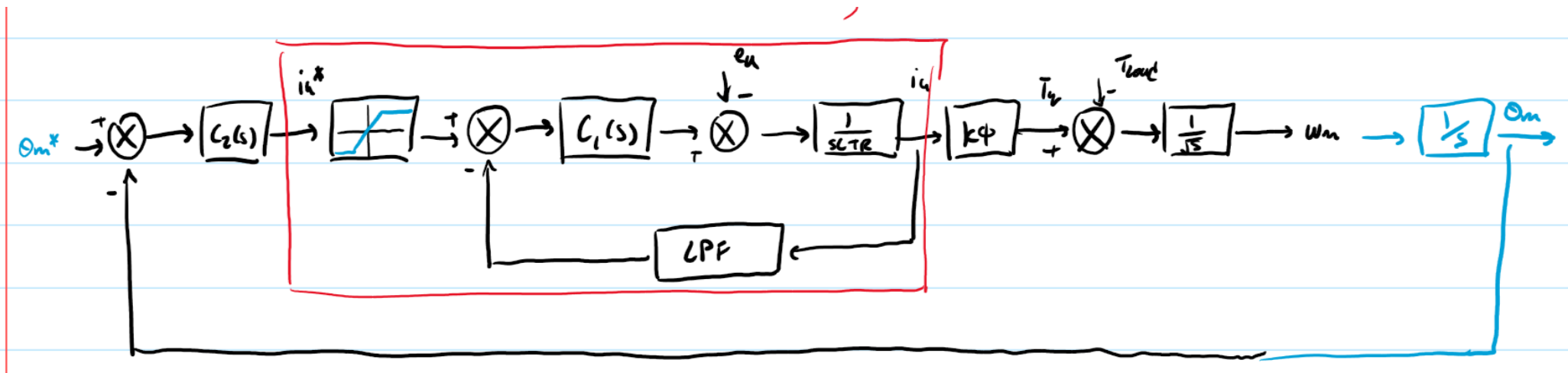


Figure 2: DC Position Control